A Simplified Kalman Estimator for an Aircraft Landing Display

ROBERT B. MERRICK*
NASA Ames Research Center, Moffett Field, Calif.

Research is being conducted on a zero-zero landing system that uses an airborne digital computer to process distance measurements from two radio transponders and a radar altimeter. It is necessary to estimate the position and velocity of the aircraft from these measurements as a preliminary to generating the pilot display, and this paper applies the Kalman filter to this task. The on-board estimator must operate within very limited allowances of computation time (70 msec) and computer storage (600 words). The pertinent observation equations are linearized around the current estimated trajectory. The first mechanization of the Kalman filter approach drastically exceeded the onboard computer constraints. Several substantial simplifications were made to meet these constraints and the results show that equivalent performance is obtainable with a much simpler system.

I. Introduction

THERE is an increasing need for the development of blind landing systems that would enable airliners to operate on schedule in all but the most extreme weather. To this end, Ames is conducting research on a zero-zero landing system that uses an airborne digital computer to process data available from distance measuring equipment and attitude angle instrumentation.[1] The necessary information is given to the pilot by an electronic display that presents an outline of the runway with the orientation and perspective that the pilot would see if the weather was good. A projected touchdown point is also included in this display.[2] This display was adequate in general, but the projected touchdown point was unacceptably noisy when it was obtained with a direct computational scheme. Accordingly, it was necessary to develop a better data processing technique.

The purpose of this paper is to describe the application of the Kalman filter to this problem and the subsequent simplification of the algorithm to meet the requirements of the onboard computer. This paper will present first the general problem and then discuss the complete Kalman filter simulation. The simplifications adopted will be explained and comparisons will be made that will point out the advantages and penalties of the simpler system.

The basic data available for computation of the aircraft position, velocity, and touchdown point consist of measurements of the distance of the airplane to three points on the ground. Transponder units are at two of these points, located symmetrically on either side of the runway. The third point is nominally underneath the airplane; thus, the third measurement is a radar altimeter reading. After appropriate processing, the data are placed on the electronic display for use by the pilot.

It is required that the data fed to the electronic display be updated frequently so that the lag introduced here will be negligible. Consequently, new data must be available at least ten times a second, and this imposes an over-all time constraint of 100 msec on the computation cycle. The utilization rate of a set of three, nominally-simultaneous, range observations is limited by the processing speed of the onboard computer, a Scientific Data Systems model 920. About 30 msec is required to update the display after estimates of the body attitude angles, inertial position, inertial velocity, etc., are available. Thus, the time allowable to process the range measurements and to obtain estimates of the position and velocity must be less than 70 msec. It is desirable, however, that this time be substantially smaller because then the over-all cycle time will be less, and certainly the pilot prefers to have his display updated as frequently as possible. Moreover, if more than a minimum of data is processed, better estimates will be obtained.

A further substantial limitation on the procedure selected for obtaining position and velocity estimates is computer storage. The memory capacity of the onboard computer is 4000 words, and 3400 words of storage are used for other purposes; so a maximum of 600 words is available for the state vector estimation.

It was expected that the Kalman filter approach to this observation and estimation problem would offer a substantial reduction in the noise of the projected touchdown point, so this filter was developed for use during a simulated landing. The pertinent observation equations were written around the current estimated trajectory and then linearized; the estimated trajectory is revised after each observation is processed.

There is also a conceptual advantage to the Kalman formulation in that a difficulty which may arise with actual hardware may be easily handled. Often when a large set of numerical measurements are examined, it is obvious that a few of these data are "blunderpoints," that is, a few of the data are wildly inconsistent with the rest of the measurements. The error signal for the Kalman filter is the difference between the expected observation and the actual observation, so gross errors are easily picked out and these data are not processed.

However, when the Kalman filter program was first written using Fortran and general purpose matrix multiplication routines, it drastically exceeded the limitations of the onboard computer. The paper will discuss the following modifications that were necessary to adapt this approach to the restrictions of the onboard computer: more efficient programming, conversion to machine language, and engineering approximations to the Kalman equations.

II. Development of the Mathematical Model

It is first necessary to discuss what equations of motion are appropriate for the landing airplane. The intended trajectory is a straight line along the glide slope until the flare is begun that is, control motions are only introduced by the pilot to null deviations from a desired letdown that has both a constant forward velocity and a constant rate of descent. This

Presented as Paper 69-944 at the AIAA Aerospace Computer Systems Conference, Los Angeles, Calif., Sept. 8–10, 1969; submitted October 27, 1969; revision received February 24, 1970.

* Research Scientist, Theoretical Guidance and Control Branch. Member AIAA.

ideal letdown also has zero lateral displacement and zero lateral velocity.

Since an accurate statistical description of the disturbing accelerations to be expected in the descent is not available and since computational simplicity is essential, these accelerations are not explicitly included in this study. The effects of these disturbances will later be treated as an increase in the variance of the estimation errors in position and velocity. Consequently, the basic equations of motion, which are used for predicting ahead the short time between observations, are taken to have zero acceleration or constant velocity in all three runway axes.

Figure 1 illustrates the runway-based, coordinate system adopted and the distances measured by the radar equipment.

The equations that relate the error-free transponder and altimeter observations (r_L, r_R, r_A) to the aircraft position are

$$\begin{aligned} r_L^2 &= x^2 + (y_0 + y)^2 + h^2 \\ r_R^2 &= x^2 + (y_0 - y)^2 + h^2 \\ r_A &= h \end{aligned} \tag{1}$$

If these equations are expanded in a Taylor Series about the estimated position $(\hat{x}, \hat{y}, \hat{h})$ and then linearized, the result is

$$r_L(x,y,h) = \hat{r}_L + (1/\hat{r}_L)[\hat{x}(x - \hat{x}) + (y_0 + \hat{y})(y - \hat{y}) + \hat{h}(h - \hat{h})]$$

$$r_R(x,y,h) = \hat{r}_R + (1/\hat{r}_R)[\hat{x}(x - \hat{x}) + (-y_0 + \hat{y})(y - \hat{y}) + \hat{h}(h - \hat{h})]$$

$$r_A = h$$

Introducing the deviation variables X and R as

$$\begin{aligned} X_1 = x - \hat{x},\ X_2 = y - \hat{y},\ X_3 = h - \hat{h} \\ X_4 = \dot{X}_1,\ X_5 = \dot{X}_2,\ X_6 = \dot{X}_3 \\ R_L = r_L - \hat{r}_L,\ R_R = r_R - \hat{r}_R,\ R_A = r_A - \hat{h} \end{aligned} \tag{2}$$

we may write the equations of motion and the linearized observation equations as

$$\begin{aligned} \dot{X}_1 = X_4,\ \dot{X}_2 = X_5,\ \dot{X}_3 = X_6 \\ \dot{X}_4 = 0,\ \dot{X}_5 = 0,\ \dot{X}_6 = 0 \end{aligned} \tag{3}$$

$$\begin{aligned} R_L &= (1/\hat{r}_L)[\hat{x}X_1 + (y_0 + \hat{y})X_2 + \hat{h}X_3] \\ R_R &= (1/\hat{r}_R)[\hat{x}X_1 + (-y_0 + \hat{y})X_2 + \hat{h}X_3] \\ R_A &= X_3 \end{aligned} \tag{4}$$

In matrix form these linear equations are

$$\underset{X(t+\mathrm{DELT})}{\begin{bmatrix} X_1(t+\mathrm{DELT}) \\ X_2(t+\mathrm{DELT}) \\ X_3(t+\mathrm{DELT}) \\ X_4(t+\mathrm{DELT}) \\ X_5(t+\mathrm{DELT}) \\ X_6(t+\mathrm{DELT}) \end{bmatrix}} = \underset{\Phi(t+\mathrm{DELT};t)}{\begin{bmatrix} 1 & 0 & 0 & \mathrm{DELT} & 0 & 0 \\ 0 & 1 & 0 & 0 & \mathrm{DELT} & 0 \\ 0 & 0 & 1 & 0 & 0 & \mathrm{DELT} \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}} \underset{X(t)}{\begin{bmatrix} X_1(t) \\ X_2(t) \\ X_3(t) \\ X_4(t) \\ X_5(t) \\ X_6(t) \end{bmatrix}} \tag{5}$$

$$\underset{R(t)}{\begin{bmatrix} R_L \\ R_R \\ R_A \end{bmatrix}} = \underset{H(t)}{\begin{bmatrix} \hat{x}/\hat{r}_L & (y_0 + \hat{y})/\hat{r}_L & \hat{h}/\hat{r}_L & 0 & 0 & 0 \\ \hat{x}/\hat{r}_R & (-y_0 + \hat{y})/\hat{r}_R & \hat{h}/\hat{r}_R & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}} \underset{X(t)}{\begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ X_6 \end{bmatrix}} \tag{6}$$

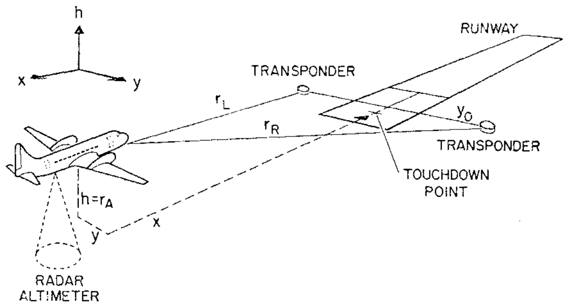

Fig. 1 Geometry and coordinates.

It is convenient to consider the 3 × 6 observation matrix as three 1 × 6 matrices; that is, consider each observation separately. This has a distinct procedural advantage because, if in a set of three, nominally-simultaneous observations there is one blunderpoint, the other two may still be processed. If the three radar observations were handled as a set, then worthwhile information would be discarded. There is also a computational advantage in considering each observation separately, since a potentially troublesome matrix inversion is replaced by three simple divisions.

Assuming that the radar measurements are corrupted only by additive noise, we may write

$$\begin{aligned} \mathrm{OBS}_L(t_{k+1}) &= r_L(t_{k+1}) + N_L(t_{k+1}) \\ \mathrm{OBS}_R(t_{k+1}) &= r_R(t_{k+1}) + N_R(t_{k+1}) \\ \mathrm{OBS}_A(t_{k+1}) &= r_A(t_{k+1}) + N_A(t_{k+1}) \end{aligned} \tag{7}$$

and

$$\begin{aligned} \mathrm{SIG}_L(t_{k+1}) &= \mathrm{OBS}_L(t_{k+1}) - \hat{r}_L(t_{k+1}) \\ &= R_L(t_{k+1}) + N_L(t_{k+1}) \\ \mathrm{SIG}_R(t_{k+1}) &= \mathrm{OBS}_R(t_{k+1}) - \hat{r}_R(t_{k+1}) \\ &= R_R(t_{k+1}) + N_R(t_{k+1}) \\ \mathrm{SIG}_A(t_{k+1}) &= \mathrm{OBS}_A(t_{k+1}) - \hat{r}_A(t_{k+1}) \\ &= R_A(t_{k+1}) + N_A(t_{k+1}) \end{aligned} \tag{8}$$

where the OBS vector is the set of radar measurements, the N vector is the added noise occurring with each measurement, and each component of the SIG vector is the error signal obtained by subtracting the expected observation from the actual observation. The right-hand side of Eq. (8) is obtained with the aid of Eqs. (2) and (7). Further, if the measurement noise is Gaussian with zero mean value and uncorrelated from one observation to the next, then applying the Kalman filter[3] to this estimation problem will produce the following perturbation variable equations:

$$\hat{X}_k = \Phi(t_{k+1};t_k)\hat{X}_k(t_k) \tag{9}$$

$$P_k = \Phi(t_{k+1};t_k)P_k(t_k)\Phi^T(t_{k+1};t_k) \tag{10}$$

$$K = P_k H^T[HP_kH^T + Q]^{-1} \tag{11}$$

$$\hat{X}_{k+1} = \hat{X}_k + K[\mathrm{SIG} - H\hat{X}_k] \tag{12}$$

$$P_{k+1} = [I - KH]P_k \tag{13}$$

Here $\hat{X}_k$ is the estimated state vector after k observations have been processed, $\Phi(t_{k+1};t_k)$ is the transition matrix, P is the covariance matrix associated with the state vector estimation errors, K is the gain matrix corresponding to a particular type of observation at a particular time, and Q is the covariance matrix of measurement errors. Unless otherwise indicated, all terms in these equations are evaluated at t_{k+1}, the time of the $k + 1$ observation.

Equations (9) and (10) change the estimate of the state vector and its covariance matrix when the new observation is taken at a time different from that of the previous observation. Equations (11–13) change the estimate of the state vector and its covariance matrix because of observational data.

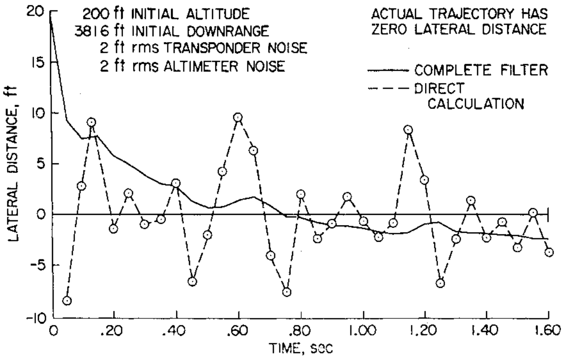

Fig. 2 Complete filter response to a position estimate error—constant velocity.

These equations are written for linear perturbations about a reference trajectory. In this instance, where the equations of motion of the airplane are very simple, it is efficient to use the estimated trajectory as the reference trajectory.[4] That is, after every observation is processed, a new estimate of position and velocity is obtained, and the linearization is accomplished about the new estimated trajectory. This means that $\hat{X}_k(t_k)$ is identically zero, so that Eq. (9) does not have to be mechanized at all and Eq. (12) is merely

$$\hat{X}_{k+1}(t_{k+1}) = K(t_{k+1})\mathrm{SIG}(t_{k+1}) \tag{14}$$

The position and velocity in runway coordinates, which is needed for the pilot display, can be obtained from the perturbation variable solution by the following relations:

$$\begin{aligned}
\hat{x}_{k+1}(t_{k+1}) &= \hat{x}_k(t_{k+1}) + \hat{X}_{1,k+1}(t_{k+1})\\
\hat{y}_{k+1}(t_{k+1}) &= \hat{y}_k(t_{k+1}) + \hat{X}_{2,k+1}(t_{k+1})\\
\hat{h}_{k+1}(t_{k+1}) &= \hat{h}_k(t_{k+1}) + \hat{X}_{3,k+1}(t_{k+1})\\
\hat{\dot{x}}_{k+1}(t_{k+1}) &= \hat{\dot{x}}_k(t_{k+1}) + \hat{X}_{4,k+1}(t_{k+1})\\
\hat{\dot{y}}_{k+1}(t_{k+1}) &= \hat{\dot{y}}_k(t_{k+1}) + \hat{X}_{5,k+1}(t_{k+1})\\
\hat{\dot{h}}_{k+1}(t_{k+1}) &= \hat{\dot{h}}_k(t_{k+1}) + \hat{X}_{6,k+1}(t_{k+1})
\end{aligned}$$

The next step in the research was to verify that this line of approach did truly have a potential for improved estimation. Accordingly, the mathematical model just developed was written in Fortran, using existing matrix multiplication routines. This mechanization did show an encouraging quick reduction in existing position errors. Figure 2 presents the response to a lateral position estimate error; this response occurs over a 200-ft interval of the downrange trajectory approximately 4000 ft from touchdown.

The results of the original direct calculation procedure are also shown on this graph. The lateral estimate obtained by this direct procedure remains noisy even in constant velocity flight, since this scheme simply solved Eq. (1) for x, y, and h in terms of the measured r_L, r_R, and r_A at each point in time. The direct procedure obtained the runway lateral velocity by applying a least-squares fit, linear or quadratic, to the last N (10–20) calculations of lateral distance.

Now that a potential for more accurate estimation had been demonstrated, it was necessary to translate this theoretical potential into a practical procedure. This was a long step because this first mechanization of the Kalman concept was far too slow (4100 msec per computation cycle compared to 60 msec available) and used three times the available 600 cells of storage. Some hope existed, however, since the laboratory computer operated in Fortran with floating point, double-precision computations, whereas the airborne computer operated in machine language with fixed point, single-precision computations. An order-of-magnitude inprovement in processing time could reasonably be expected from this change alone, and further improvements were available through efficient programming and use of justifiable engineering simplifications. It was felt that the single-precision accuracy (24 binary bits + 1 parity bit) would be accurate enough if carefully programmed.

III. Immediate Simplifications

In many engineering studies separation of variables or modes of motion offers great simplification and has solid justification; the lateral and longitudinal modes of motion of aircraft are often studied separately. In this study an examination of the observation equations [Eq. (6)] was encouraging, since the altitude is observed separately from the lateral and longitudinal position coordinates. It was also expected that the measurement errors of the altimeter would be less than the errors in the transponder measurements, so altitude would be well determined. Accordingly it seemed reasonable that the errors in altitude and altitude rate at any particular time would have negligible correlation with the errors in x, $\dot{x}$, y, and $\dot{y}$ at the same time and this assumption was adopted. This first assumption inserts 16 zeros into the 36-element, covariance matrix of estimation errors (P matrix); and, since the covariance matrix is involved in all of Eqs. (9–13), this assumption has major effect.

The next direct simplification was to take advantage of the many zeros now in the P matrix and in the H matrix and to capitalize also on both the zeros and ones in the transition matrix. That is, all multiplications by zero or one can be eliminated from the iterative cycle by writing down all matrix multiplications in complete detail.

While implementing the previous simplification and first assumption in connection with the covariance matrix updating equation [Eq. (10)], it was observed that there were four elements in the updated P matrix that had a term involving a power of the time increment. Each of these four elements could be written as:

$$\begin{aligned}
P_{i,j}\text{ updated} &= P_{i,j} + (P_{i+3,j+3})(\mathrm{DELT})^2\\
&\quad + (P_{i,j+3} + P_{i+3,j})(\mathrm{DELT})
\end{aligned}$$

With DELT < 0.10 sec, the $(\mathrm{DELT})^2$ terms could be safely dropped, and this became the second simplifying assumption.

The Fortran program was altered to incorporate these improvements, and it was found that the associated performance penalty was negligible. A complete listing of the equations at this point is given in the Appendix.

The time per computation cycle was now down to 320 msec while still in Fortran, floating point, double precision. An order-of-magnitude improvement had already been obtained, and the change to machine language, fixed point, single precision was still to come, so it was clear that the basic time requirement could be met. Actually, after further simplifications, the time per computation cycle of the final machine language filter was 27 msec.

IV. Computational Difficulties and Further Simplification

When optimal theories are applied to physical problems computational difficulties may occur.[5] These difficulties should be expected whenever the mathematical model selected to represent the physical world is not a good description of the actual dynamics and geometry or when the model selected is a good, but not perfect, description and many data are to be processed. In this application, the model is known to be imperfect since gust disturbances have not been included, and triads of radar data will be processed many times a second for perhaps more than 200 sec.

If the mathematical model in the Appendix were used, one would expect the covariance matrix of estimation errors to shrink gradually as more data are processed and to approach zero until it violates the theoretical requirement that it be a positive definite matrix. This theoretical anomaly would occur because only a limited number of significant figures can be used in the computations and extreme accuracy is required when the terms in the covariance matrix become very small.

However, a physical anomaly would arise long before the computation difficulty could cause trouble. With gust disturbances, winds shifting direction with altitude, four only nominally-identical engines, etc., how well can we expect to know the velocity of the airplane from frequent position measurements?

Consider a one-quarter g disturbance lasting half a second; the deviations resulting at the end of this disturbance are 4.0 ft/sec in velocity, but only 1.0 ft in position. Clearly, this position cannot be reliably sensed with half a dozen measurements using distance measuring equipment whose rms error is twice the position deviation. This physical reasoning implies that the variance of the estimation error in the three components of velocity should never be much less than $(4.0\ \text{ft/sec})^2$. This also establishes some minima for the variances of the estimation error in the three components of position.

It is clear that the equations in the Appendix must be altered so that these known physical characteristics are more adequately represented, but increased computational complexity is undesirable. Consider the major simplification of letting the velocity variances be constant and equal to the previously discussed minima. This would handicap the performance of the estimation system in the initial portion of the approach, since, at that time, the actual errors in velocity estimation may be substantially greater than the selected minima and error reduction would be slow. However, we may quite tolerably allow this initial error reduction phase to be as long as 30 sec and still have about 3 min of accurate estimation before landing. Accordingly, this major simplification was adopted.

The calculation of the variances for the estimation error in position cannot be simplified quite as drastically because the initial phase of the approach needs reasonable speed in reducing position error in the first few seconds of operation. Consequently, these variances must be large initially for quick response, but still must be prevented from becoming unrealistically small. The solution adopted here was to add a small constant to each of the three equations for the time updating of the variances of the estimation error for runway position. That is, as every set of data was processed, each of the three variances was artificially increased a small amount. This modification and the velocity variance simplification just discussed ensure that the overall estimation system retains significant sensitivity to new observations; consequently, recent data contribute more to the current estimate than older data.

The performance of the system with these new modifications incorporated was evaluated and found to be satisfactory.

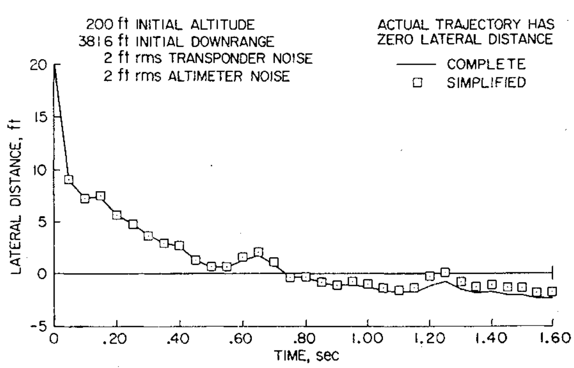

Fig. 3 Comparison of the complete and simplified filters—constant velocity.

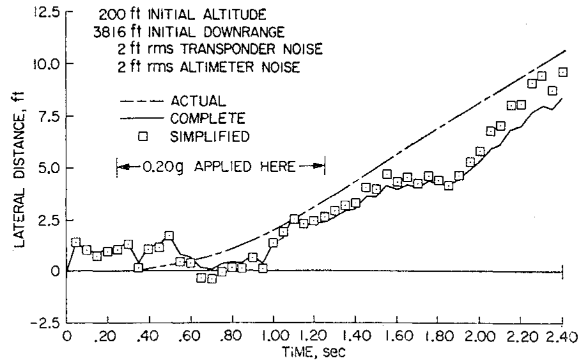

Fig. 4 Comparison of the complete and simplified filters—response to a lateral gust.

During this evaluation it was empirically determined that a few of the remaining cross-correlation terms of the covariance matrix were not significant and they were dropped. An example of such a term is the correlation between the error in estimate of the downrange position and the error in estimate of the lateral velocity.

After all simplifications had been completed, the final system of equations became as follows:

The equations that update the covariance matrix and the airplane position for the time between observations are

$$P11 = P11 + 2.0(\text{DELT})(P14) + 0.6,\quad P44 = 16.0$$
$$P14 = P14 + (\text{DELT})(P44),\quad P55 = 16.0$$
$$P22 = P22 + 2.0(\text{DELT})(P25) + 0.3,\quad P66 = 9.0$$
$$P25 = P25 + (\text{DELT})(P55),\quad P33 = P33 + 2.0(\text{DELT})(P36) + 0.4$$
$$P36 = P36 + (\text{DELT})(P66),\quad \hat{x}(t_{k+1}) = \hat{x}(t_k) + \text{DELT}[\hat{\dot{x}}(t_k)]$$
$$\hat{y}(t_{k+1}) = \hat{y}(t_k) + \text{DELT}[\hat{\dot{y}}(t_k)],\quad \hat{h}(t_{k+1}) = \hat{h}(t_k) + \text{DELT}[\hat{\dot{h}}(t_k)]$$

The following equations use the error signal and observation matrix associated with observation M to improve the estimate of the state vector and the error covariance matrix:

$$PHT(1) = P11[H(1)] + P12[H(2)]$$
$$PHT(2) = P22[H(2)] + P12[H(1)]$$
$$PHT(3) = P33[H(3)],\quad PHT(4) = P14[H(1)]$$
$$PHT(5) = P25[H(2)],\quad PHT(6) = P36[H(3)]$$
$$HPHT = H(1)[PHT(1)] + H(2)[PHT(2)] + H(3)[PHT(3)]$$
$$DU12 = 1.0/[HPHT + Q(M)]$$
$$GD(\text{I}) = DU12[PHT(\text{I})],\quad X(I) = X(I) + GD(I)[\text{SIG}(M)] \quad \text{for } I = 1,6$$

$$P11 = P11 - GD(1)[PHT(1)]$$
$$P12 = P12 - GD(1)[PHT(2)]$$
$$P14 = P14 - GD(1)[PHT(4)]$$
$$P22 = P22 - GD(2)[PHT(2)]$$
$$P25 = P25 - GD(2)[PHT(5)]$$
$$P33 = P33 - GD(3)[PHT(3)]$$
$$P36 = P36 - GD(3)[PHT(6)]$$

The error signals and observation matrices used here are identical to those in the Appendix.

V. Filter Comparisons

This far simpler estimation system has substantially the same performance as the full filter. Figure 3 shows a direct comparison between the Fortran versions of the two filters as they respond to an initial error in estimate of lateral distance; all initial conditions are identical, the observation noise is identical, and the responses are almost identical.

Lateral gust responses for the same two systems are presented in Fig. 4, and again the responses show that the sim-

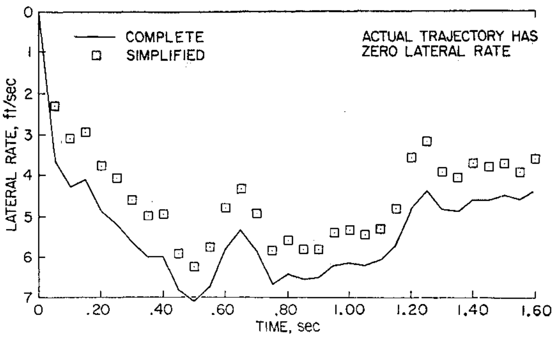

Fig. 5 Comparison of the complete and simplified filters—constant velocity.

plified system is equivalent in performance to the complete filter. Figures 5 and 6 show the lateral rate responses associated with Figs. 3 and 4, and again it is clear that the major simplifications adopted were justified since equivalent performance is demonstrated. The simplified filter actually outperforms the complete filter as time passes because then it has more sensitivity to additional data.

The prediction of the impact point was much improved using this simple filter. The impact point, determined by the ratio of lateral velocity to downrange velocity, had been far too jumpy when predicted by the direct procedure and frequently jumped out of the range of the pilot display. Now the jumpiness was tolerable, although the velocity ratio did not always stay within the desired error limit of 0.01.

After the simplified system was programmed in machine language, fixed point, single precision, it was necessary to verify that accuracy or overflow problems did not occur, although the Kalman formulation has a natural inclination to avoid overflow difficulties. This is so because the error signal to the filter is a perturbation quantity, and, consequently, the magnitude of the input to the filter is nominally unaffected by the magnitude variations of basic physical quantities such as range to touchdown, etc.

Consequently, several runs, which had a wide range of initial conditions and variable size, were compared, and it was determined that results were satisfactory. The accuracy loss was, in effect, smothered by the small constants that were added to ensure that the variances of estimation errors in position had realistic minima; that is, the accuracy loss was of much less effect than the small constants in those particular equations. The only place that accuracy was critical was in calculating the estimated observations $\hat{r}_L, \hat{r}_R$ where a double-precision, square root routine was required.

VI. Conclusions

The Kalman filter equations were developed for use in a zero-zero landing system because the pilot display was not satisfactory due to inaccuracies in position and velocity estimation when a direct procedure was used. The first computer simulation of the Kalman approach gave substantial improvement in performance, but was far too slow and used too much computer space. A much simplified version of this filter has been obtained that has performance equivalent to the complete filter and does meet the onboard computer requirements. The machine language version of the simplified filter completes a cycle of computations for three, nominally-simultaneous, distance measurements in 27 msec and uses less than 500 words of computer storage.

Minima for the variances of estimation error for position and velocity were built into the mathematical model in an extremely simple manner. Flight tests have shown much improvement in the estimation of the impact point although the response time is not as fast as desired.

VII. Appendix: Equations that Result from Using the First Two Simplifications

The equations for the time updating of the covariance matrix of estimation errors are

$$P11 = P11 + 2\,\mathrm{DELT}(P14),\ P12 = P12 + \mathrm{DELT}(P15 + P24)$$
$$P14 = P14 + \mathrm{DELT}(P44),\ P15 = P15 + \mathrm{DELT}(P45)$$
$$P22 = P22 + 2\,\mathrm{DELT}(P25),\ P24 = P24 + \mathrm{DELT}\,(P45)$$
$$P25 = P25 + \mathrm{DELT}(P55),\ P33 = P33 + 2\,\mathrm{DELT}(P36)$$
$$P36 = P36 + \mathrm{DELT}(P66),\ P44 = P44,\ P45 = P45$$
$$P55 = P55,\ P66 = P66$$

Since the P matrix is symmetric, only terms on or above the main diagonal are listed. Those not listed are zero.

The equations for the time updating of the airplane position are

$$\hat{x}(t_{k+1}) = \hat{x}(t_k) + \mathrm{DELT}[\hat{\dot{x}}(t_k)]$$
$$\hat{y}(t_{k+1}) = \hat{y}(t_k) + \mathrm{DELT}[\hat{\dot{y}}(t_k)]$$
$$\hat{h}(t_{k+1}) = \hat{h}(t_k) + \mathrm{DELT}[\hat{\dot{h}}(t_k)]$$

The equations that generate the error signal, SIG, and the observation matrix, H, associated with the Mth observation are $(M = 1,2,3)$

$$\hat{r}_L = SQRT[\hat{x}^2 + \{y_0 + \hat{y}\}^2 + \hat{h}^2],\ \mathrm{SIG}_L = \mathrm{OBS}_L - \hat{r}_L$$
$$H(1) = \hat{x}/\hat{r}_L,\ H(2) = [y_0 + \hat{y}]/r_L$$
$$H(3) = \hat{h}/\hat{r}_L,\ \hat{r}_R = SQRT[\hat{x}^2 + \{y_0 - \hat{y}\}^2 + \hat{h}^2]$$
$$\mathrm{SIG}_R = \mathrm{OBS}_R - \hat{r}_R,\ H(1) = \hat{x}/\hat{r}_R$$
$$H(2) = [-y_0 + \hat{y}]/\hat{r}_R,\ H(3) = \hat{h}/\hat{r}_R$$
$$\mathrm{SIG}_A = \mathrm{OBS}_A - \hat{h},\ H(1) = 0.0$$
$$H(2) = 0.0,\ H(3) = 1.0$$

The next equations calculate the gain matrix, GD, associated with the particular observation matrix, H.

$$PHT(1) = P11[H(1)] + P12[H(2)],PHT\ (2) = P22[H(2)] + P12[H(1)]$$
$$PHT(3) = P33[H(3)],\ PHT(4) = P14[H(1)] + P24[H(2)]$$
$$PHT(5) = P15[H(1)] + P25[H(2)],\ PHT(6) = P36[H(3)]$$
$$HPHT = H(1)[PHT(1)] + H(2)[PHT(2)] + H(3)[PHT(3)]$$
$$DU12 = 1.0/[HPHT + Q(M)],\ GD(I) = PHT(I)(DU12)\ \text{for}\ I = 1{,}6$$

Next, the estimate of the state vector is altered, using the information of observation M.

$$X(I) = X(I) + GD(I)[\mathrm{SIG}(M)]$$

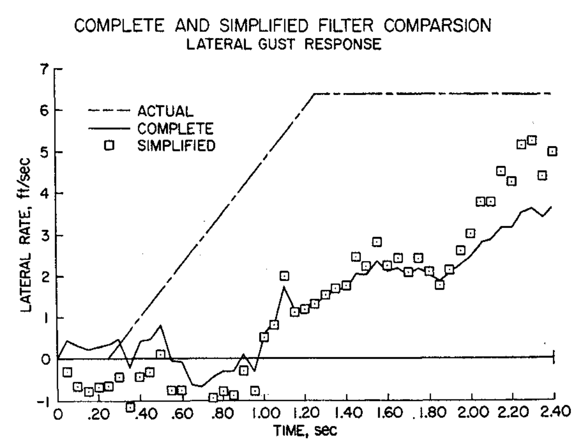

Fig. 6 Comparison of the complete and simplified filters—response to a lateral gust.

This final set of equations recomputes the covariance matrix. It indirectly uses all the data except the current observation.

$$P11 = P11 - GD(1)[PHT(1)], P12 = P12 - GD(1)[PHT(2)]$$

$$P14 = P14 - GD(1)[PHT(4)], P15 = P15 - GD(1)[PHT(5)]$$

$$P22 = P22 - GD(2)[PHT(2)], P24 = P24 - GD(2)[PHT(4)]$$

$$P25 = P25 - GD(2)[PHT(5)]$$

$$P33 = P33 - GD(3)[PHT(3)], P36 = P36 - GD(3)[PHT(6)]$$

$$P44 = P44 - GD(4)[PHT(4)], P45 = P45 - GD(4)[PHT(5)]$$

$$P55 = P55 - GD(5)[PHT(5)], P66 = P66 - GD(6)[PHT(6)]$$

VIII. References

[1] Robinson, G. G. and Johnson, N. S., "Subsystem Requirements for an Airborne Laboratory to Study Zero-Zero Landing Systems." Rept. 488, Oct. 1964, NATO Advisory Group for Aeronautical Research and Development.

[2] Douvillier, J. G. and Foster, J. V., "Research on a Manually Piloted Airborne Zero-Zero Landing System," 15th International Air Transport Association Technical Conference on All Weather Landing and Takeoff, Lucerne, Switzerland, April 25–May 4, 1963.

[3] Smith, G. L., Schmidt, S. F., and McGee, L. A., "Applications of Statistical Filter Theory to the Optical Estimation of Position and Velocity Onboard a Circumlunar Vehicle," TR R-135, 1962, NASA.

[4] Carney, T. M. and Goldwyn, R. M., "Construction and Numerical Evaluation of a Group of Estimators for a Joint State and Parameter Identification in Autonomous Linear Systems," Series EE66 No. 19, Grant No. GU-1153, Sept. 1966, Dept. of Electrical Engineering, Rice University, Houston, Texas.

[5] Schlee, F. H., Standish, C. J., and Toda, N. F., "Divergence in the Kalman Filter," *AIAA Journal*, Vol. 5, No. 6, June 1967, pp. 1114–1120.